Note: These questions will be discussed in the tutorial sessions on October 10th 2014.

Question 1:

(a) Describe the substitutional and interstitial diffusion mechanisms in solid metals

(b) Compare interstitial and vacancy atomic mechanisms for diffusion.

(c) Cite two reasons why interstitial diffusion is normally more rapid than vacancy diffusion.

Solution

(a) During the substitutional diffusion of atoms in a solid alloy crystal lattice, solute atoms move into positions of solvent atoms in the matrix through a vacancy mechanism. In interstitial diffusion, small solute atoms move between the interstices of the solvent lattice.

(b) For the interstitial diffusion mechanism, atomic motion is from interstitial site to adjacent interstitial site. On the other hand, with vacancy diffusion, atomic motion is from one lattice site to an adjacent vacancy. Self-diffusion and the diffusion of substitutional impurities proceed via this mechanism.

(c) Interstitial diffusion is normally more rapid than vacancy diffusion because: (1) interstitial atoms, being smaller, are more mobile; and (2) the probability of an empty adjacent interstitial site is greater than for a vacancy adjacent to a host (or substitutional impurity) atom.

Question 2:

A sheet of steel 1.5 mm thick has nitrogen atmospheres on both sides at 1200℃ and is permitted to achieve a steady-state diffusion condition. Knowing that the concentration of nitrogen in the steel at the high-pressure surface is 4 kg/m^3, how far into the sheet from this high-pressure side will the concentration be 2.5 kg/m^3? The diffusion coefficient for nitrogen in steel at this temperature is 6×10^{-11} m^2/s, the diffusion flux is 1.2×10^{-7} kg/m^2-s and the concentration profile is linear.

Solution

Equation 5.3

$$J = -D\frac{C_A - C_B}{x_A - x_B}$$

C_A : the point at which the concentration of nitrogen is 4 kg/m3

$$x_B = x_A + D\left[\frac{C_A - C_B}{J}\right] = 0 + (6\times10^{-11}\ \mathrm{m^2/s})\left[\frac{4\,kg/m^3 - 2.5\,kg/m^3}{1.2\times10^{-7}\,kg/m^2\text{-}s}\right] = 7.5\times 10\text{-}4\ m = 0.75\ mm$$

Question 3:

N_2 (gas) is to be diffused into pure iron at 700℃. What will be the concentration 1 mm from the surface after 10 h if the surface concentration is maintained at 0.15 wt% N. The diffusion coefficient for nitrogen in iron at 700℃ is 2.5×10^{-11} m^2/s.

Solution

Equation 5.5 (Nonsteady-state diffusion

$$\frac{C_x - C_0}{C_s - C_0} = \frac{C_x - 0}{0.15 - 0} = 1 - \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

$$= 1 - \mathrm{erf}\left[\frac{10^{-3}\,m}{(2)\sqrt{(2.5\times10^{-11}\mathrm{m^2/s})(10h)(3600s/h)}}\right] = 1 - \mathrm{erf}\,(0.527)$$

Using data in Table 5.1 and linear interpolation

z	*erf (z)*
0.500	*0.5205*
0.527	*y*
0.550	*0.5633*

$$\frac{0.527 - 0.500}{0.550 - 0.500} = \frac{y - 0.5205}{0.5633 - 0.5205} \quad \Rightarrow \quad y = erf\,(0.527) = 0.5436$$

Finally,

$$\frac{C_x - 0}{0.15 - 0} = 1 - 0.5436 \quad \Rightarrow \quad C_x = 0.068\ wt\%\ N$$

Question 4:

The diffusion coefficients for iron in nickel are given at two temperatures:

$T\ (K)$	$D\ (m^2/s)$
1273	9.4×10^{-16}
1473	2.4×10^{-14}

(a) Determine the values of D_0 and the activation energy Q_d.

(b) What is the magnitude of D at 1100ºC (1373 K)?

Solution

(a) From Equation 5.9a,

$$\ln D_1 = \ln D_0 - \frac{Q_d}{R}\left(\frac{1}{T_1}\right)$$

$$\ln D_2 = \ln D_0 - \frac{Q_d}{R}\left(\frac{1}{T_2}\right)$$

Now, solving for Q_d

$$Q_d = -R\,\frac{\ln D_1 - \ln D_2}{\dfrac{1}{T_1} - \dfrac{1}{T_2}} = -\,(8.31\ \text{J/mol-K})\,\frac{\left[\ln\left(9.4 \times 10^{-16}\right) - \ln\left(2.4 \times 10^{-14}\right)\right]}{\dfrac{1}{1273\ \text{K}} - \dfrac{1}{1473\ \text{K}}} = 252{,}400\ \text{J/mol}$$

Now, solving for D_0 from Equation 5.8

$$D_0 = D_1 \exp\left(\frac{Q_d}{RT_1}\right) = \left(9.4 \times 10^{-16}\ \text{m}^2/\text{s}\right)\exp\left[\frac{252{,}400\ \text{J/mol}}{(8.31\ \text{J/mol-K})(1273\ \text{K})}\right] = 2.2 \times 10^{-5}\ \text{m}^2/\text{s}$$

(b)

$$D = \left(2.2 \times 10^{-5}\ \text{m}^2/\text{s}\right)\exp\left[-\frac{252{,}400\ \text{J/mol}}{(8.31\ \text{J/mol-K})(1373\ \text{K})}\right] = 5.4 \times 10^{-15}\ \text{m}^2/\text{s}$$

Question 5:

A cylindrical copper rod (E = 110 GPa) is to be subjected to a load of 6660 N. If the length of the rod is 400 mm, what must be the diameter to allow an elongation of 0.50 mm? Yield strength of copper is 240 MPa.

Solution

Employing Equations 6.1, 6.2, and 6.5, assuming that deformation is entirely elastic

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi\left(\frac{d_0^2}{4}\right)} = E\,\frac{\Delta l}{l_0}$$

$$d_0 = \sqrt{\frac{4\,l_0 F}{\pi E \Delta l}} = \sqrt{\frac{(4)(400\times10^{-3}\,m)(6660\,N)}{(\pi)(110\times10^{9}\,N/m^2)(0.50\times10^{-3}\,m)}} = 7.85 \times 10^{-3}\text{ m} = 7.85\text{ mm}$$

Question 6:

A cylindrical specimen of some alloy (E=140 GPa) 8 mm in diameter is stressed elastically in tension. A force of 15,100 N produces a reduction in specimen diameter of 5 × 10^{-3} mm. Calculate Poisson's ratio for this material.

Solution

Equations 6.5 and 6.1

$$\varepsilon_z = \frac{\sigma}{E} = \frac{F}{A_0 E} = \frac{F}{\pi\left(\frac{d_0}{2}\right)^2 E} = \frac{4F}{\pi d_0^2 E}$$

Transverse strain εx

$$\varepsilon_x = \frac{\Delta d}{d_0}$$

Poisson's ratio (Equation 6.8)

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\Delta d / d_0}{\left(\frac{4F}{\pi d_0^2 E}\right)} = -\frac{d_0 \Delta d \pi E}{4F}$$

$$= -\frac{(8\times10^{-3}\,m)(-5\times10^{-6}\,m)(\pi)(140\times10^{9}\,N/m^2)}{(4)(15{,}100\,N)} = 0.291$$

Question 7:

T*he following true stresses produce the corresponding true plastic strains for a brass alloy:*

True Stress (psi)	True Strain
50,000	0.10
60,000	0.20

What true stress is necessary to produce a true plastic strain of 0.25?

Solution

Equation 6.19 $$\sigma_T = K(\varepsilon_T)^n$$

$$\log(50{,}000\ \text{psi}) = \log K + n \log(0.10)$$

$$\log(60{,}000\ \text{psi}) = \log K + n \log(0.20)$$

$$n = \frac{\log(50{,}000) - \log(60{,}000)}{\log(0.10) - \log(0.20)} = 0.263$$

and

$$\log K = 4.96 \quad \Rightarrow \quad K = 10^{4.96} = 91{,}623\ \text{psi}$$

$\varepsilon_T = 0.25$ $\Rightarrow$ $$\sigma_T = K(\varepsilon_T)^n = (91{,}623\ \text{psi})(0.25)^{0.263} = 63{,}700\ \text{psi}\ \ (440\ \text{MPa})$$